Stellar streams and dark substructure

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with Fabian Schmidt

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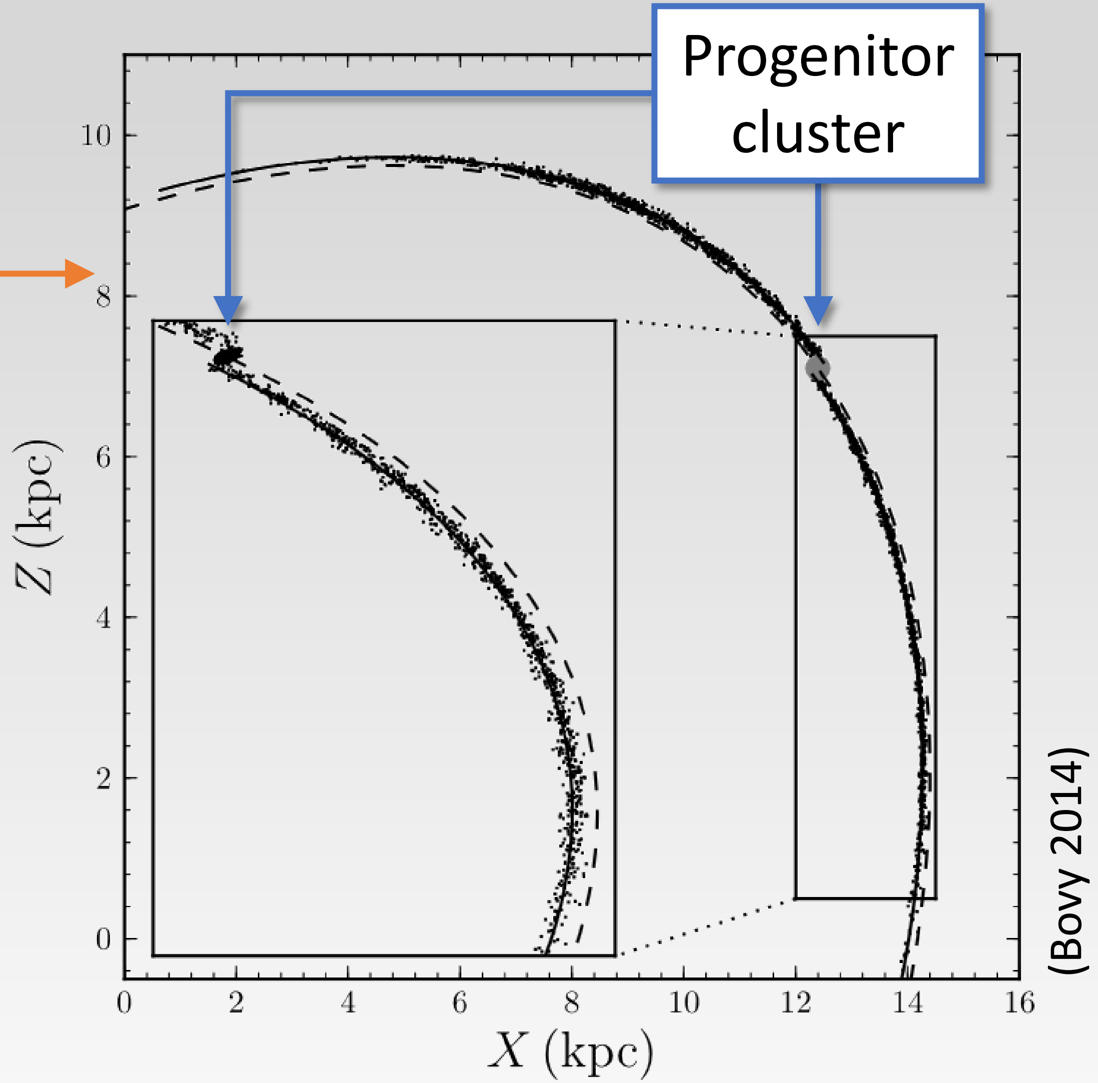
Stellar streams

Stars tidally stripped from a globular cluster or dwarf galaxy form a stellar stream



Stellar streams have negligible self-gravity and are dynamically cold

↓

They are sensitive to, and retain a sharp memory of, any gravitational perturbations

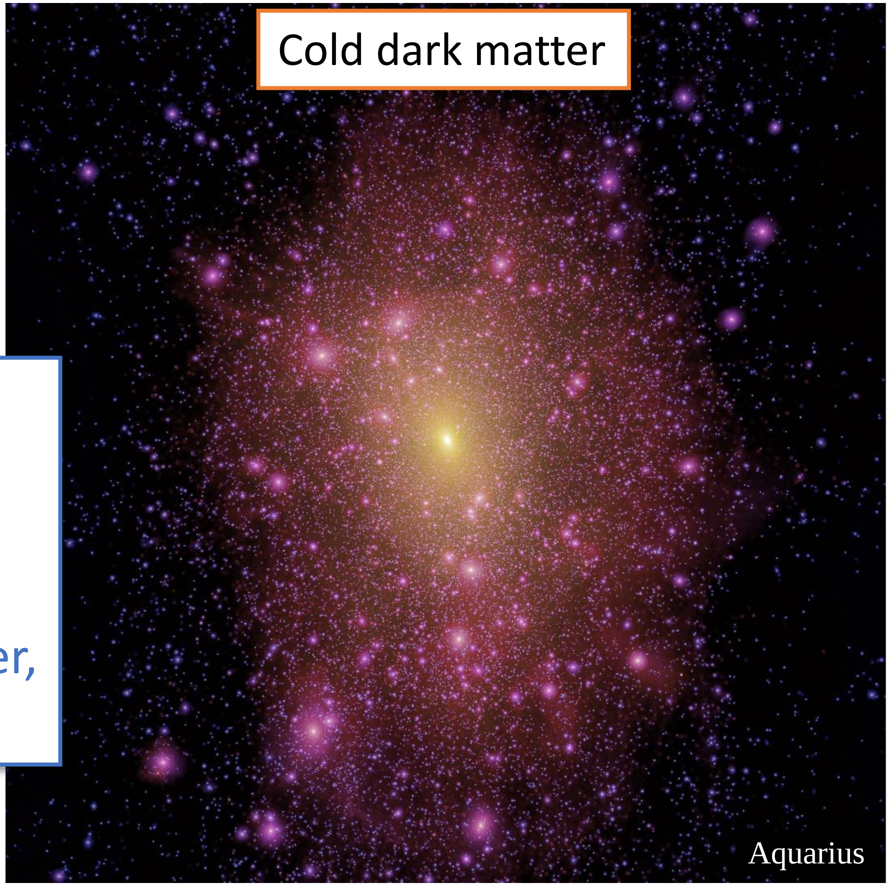
Dark matter substructure

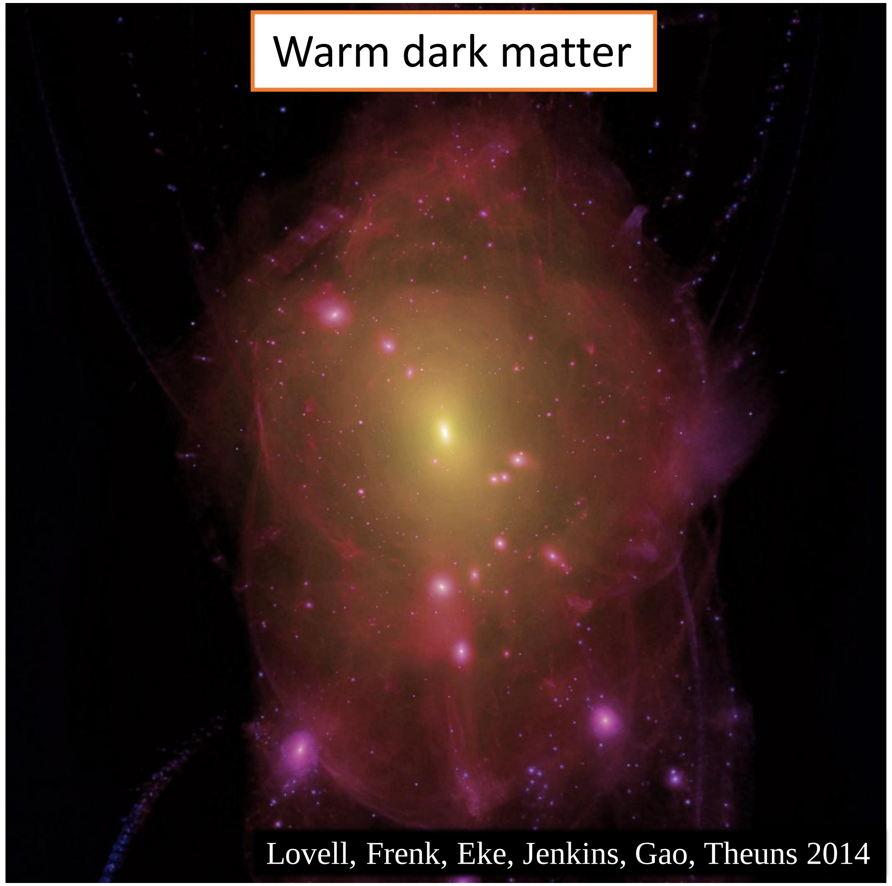
Properties of dark matter substructure can probe the properties of dark matter.

Example:
CDM vs WDM

Only the largest CDM subhalos contain stars.

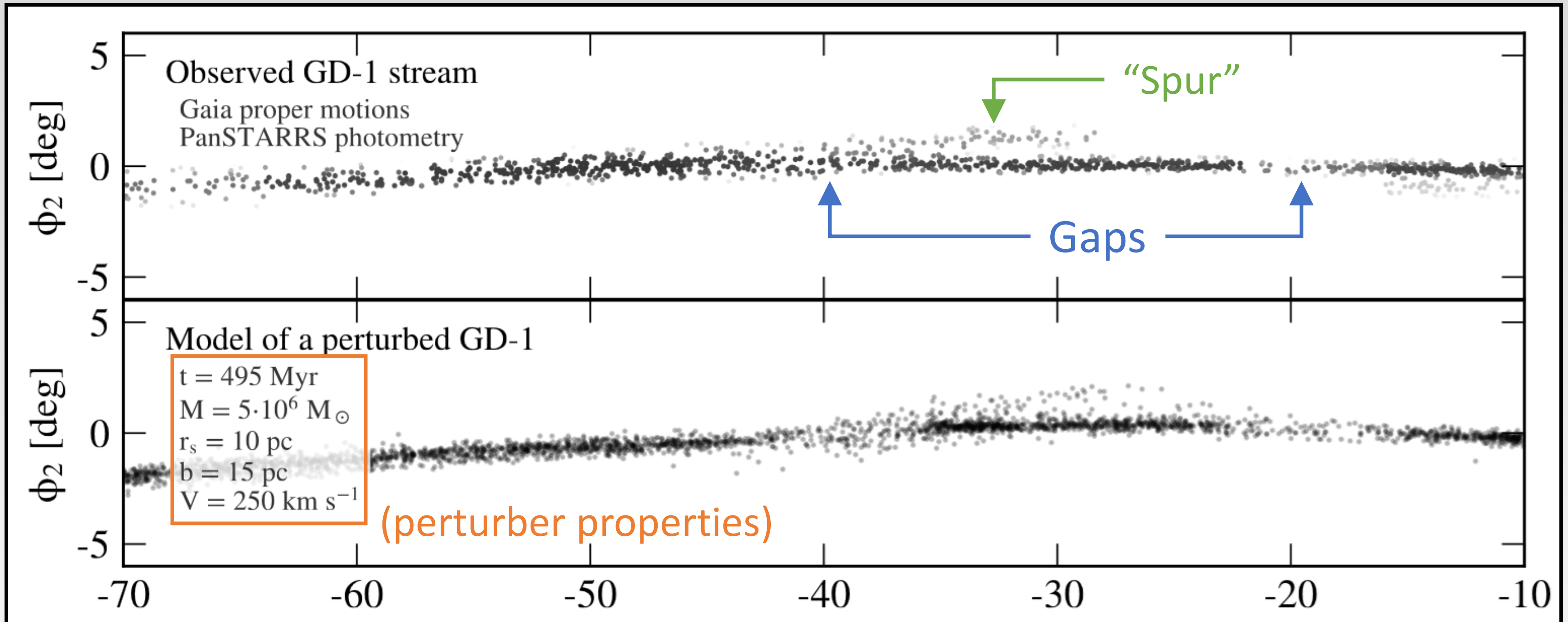
Can we use stellar streams to detect smaller, entirely dark subhalos?





Evidence for substructure encounters

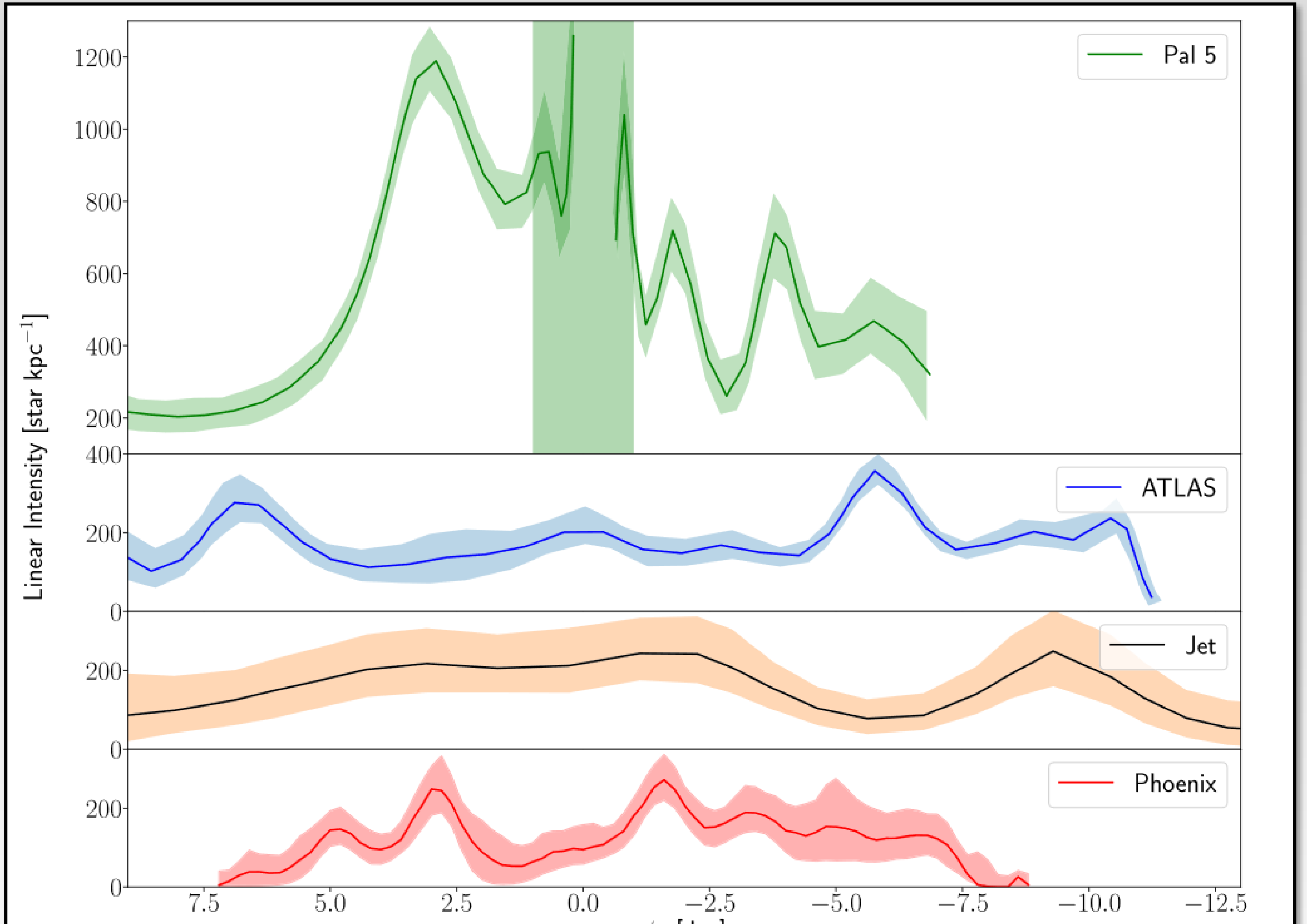
Features have been noted in stellar streams that may indicate past subhalo encounters.



Bonaca, Hogg, Price-Whelan, Conroy 2019

Stream density variations

Several other streams are also known to exhibit significant density variations:

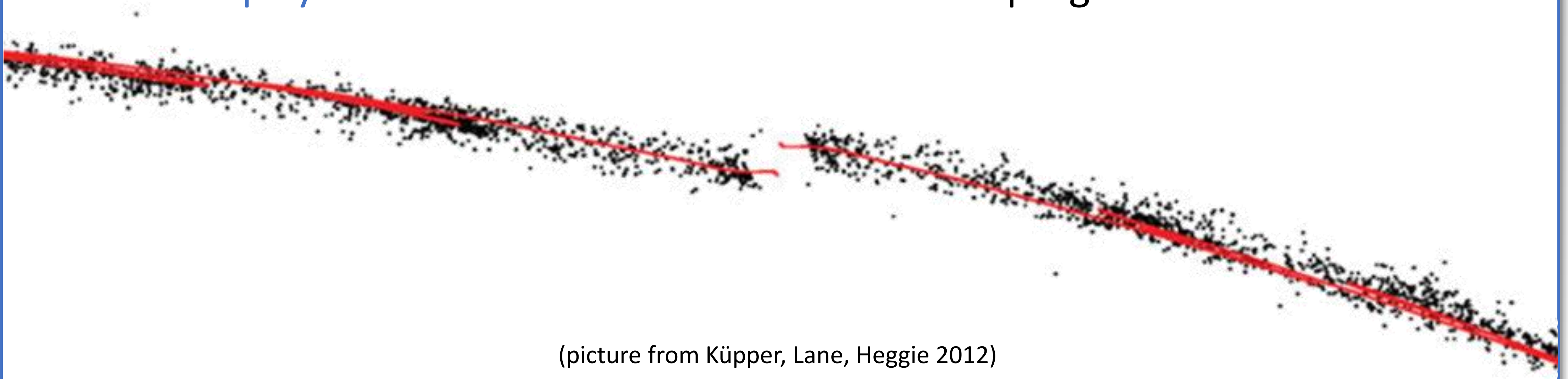


Tavangar et al. 2022

Causes for stream density variations

Not necessarily an indication of dark matter substructure

Streams are naturally inhomogeneous due to
epicyclic motion of stars released from the progenitor cluster.



(picture from Küpper, Lane, Heggie 2012)

Effect may be suppressed by the stream's velocity dispersion as it ages (Banik et al. 2021)

Causes for stream density variations

Not necessarily an indication of dark matter substructure

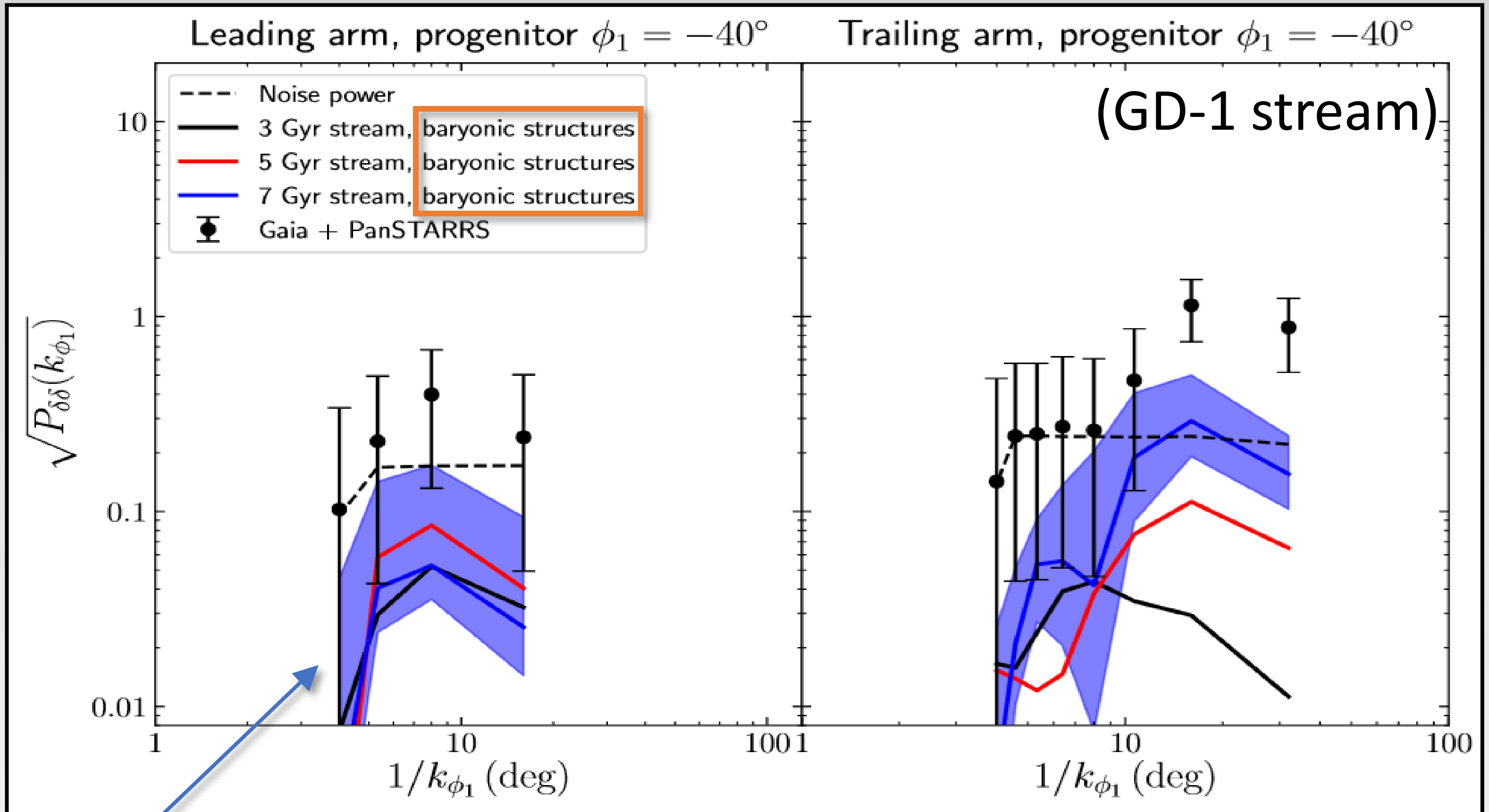
Stream could be perturbed by
baryonic substructure:

- Milky way galaxy: bar and spiral arms
- Globular clusters and giant molecular clouds
- Satellite galaxies
  [Particularly relevant if a stream's progenitor accreted together with its previous host: many low-speed encounters]

Impact of baryonic structures is suppressed for retrograde streams (like GD-1) due to faster encounters.

Stream power spectrum

Instead of searching for individual features, consider the power spectrum of the stream's linear density:



Banik, Bovy, Bertone, Erkal, de Boer 2021

Prediction using numerical simulations

Velocity-kick correlations

Idea: consider correlations between velocity kicks $\Delta\boldsymbol{v}$ at different positions.

Given a density field $\rho(\boldsymbol{x}) = \bar{\rho}\delta(\boldsymbol{x})$ moving at relative velocity $\boldsymbol{u}$, write $\Delta\boldsymbol{v}$ as an integral over Fourier space:

$$\Delta\boldsymbol{v}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\delta(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{V}^*(\boldsymbol{k}|\boldsymbol{u},t)$$

Depends on $\bar{\rho}$, $\boldsymbol{k}$, $\boldsymbol{u}$, and time t: $\boldsymbol{V}(\boldsymbol{k}|\boldsymbol{u},t) \equiv 8\pi\mathrm{i}G\bar{\rho}\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{u}t/2}\frac{\sin(\boldsymbol{k}\cdot\boldsymbol{u}\,t/2)}{\boldsymbol{k}\cdot\boldsymbol{u}}\frac{\boldsymbol{k}}{k^2}$

Then construct the power spectrum:

$$P_{\hat{\boldsymbol{a}}\hat{\boldsymbol{b}}}(k) \equiv \int_{-\infty}^{\infty}\mathrm{d}r\,\mathrm{e}^{-\mathrm{i}kr}\langle\hat{\boldsymbol{a}}\cdot\Delta\boldsymbol{v}(0)\hat{\boldsymbol{b}}\cdot\Delta\boldsymbol{v}(\boldsymbol{r})\rangle$$

Velocity-kick power spectrum

…after much work we obtain:

Dimensionless density power spectrum of perturbing environment (baryons + DM)

Substructure velocity distribution (now in the Galactic frame)

$$P_{\Delta v}(k,t) = 16\pi^4 G^2 \bar{\rho}^2 k^2 t \int_k^\infty \frac{\mathrm{d}q}{q} \frac{\mathcal{P}(q)}{q^6} \int \mathrm{d}^3\boldsymbol{u} \frac{f(\boldsymbol{u})}{u} \theta_H(qu - kv)$$

Power spectrum of $\Delta \boldsymbol{v}$ along the stream

Stream velocity

Important consequence: stream perturbations at scale k only arise from substructure at smaller scales $q > k$.

Response of the stream to $P_{\Delta v}(k)$

Approximate the stellar stream as a one-dimensional system with no self-gravity. Let $f(x, v, t)$ be its DF.

Boltzmann equation:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -C\frac{\partial f}{\partial v} + \frac{1}{2}D\frac{\partial^2 f}{\partial v^2}$$

(Fokker-Planck approximation)

Diffusion coefficients:

$C(x,t) = \frac{\Delta v}{\Delta t}$ (injection rate of coherent velocities)

$D(x,t) = \frac{(\Delta v)^2}{\Delta t}$ (injection rate of random motion)

[Δv = velocity kick per time Δt]

Idea: relate $P_{\Delta v}(k)$ to the statistics of $C(x,t)$ and $D(x,t)$.

Perturbative treatment

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -C\frac{\partial f}{\partial v} + \frac{1}{2}D\frac{\partial^2 f}{\partial v^2}$$

- Expand $f(x,v,t) = f_0(v,t) + f_1(x,v,t)$
- Assume $C(x,t) = \frac{\Delta v}{\Delta t}$ is perturbative
- Assume $D = \frac{(\Delta v)^2}{\Delta t}$ is spatially uniform (not ideal)

Equation for spatial average:

$$\frac{\partial f_0}{\partial t} = \frac{1}{2}D\frac{\partial^2 f_0}{\partial v^2} \implies \sigma^2(t) = \sigma_0^2 + Dt$$

Equation for first-order perturbations (in Fourier space):

$$\frac{\partial f_1(k,v,t)}{\partial t} + ikv f_1(k,v,t) = -C(k,t)\frac{\partial f_0(v,t)}{\partial v} + \frac{1}{2}D\frac{\partial^2 f_1(k,v,t)}{\partial v^2}$$

Approach:

- Solve for $f_1(k,v,t)$
- Integrate over velocities to obtain $\delta_*(k,t)$
- Evaluate the correlation function $\langle \delta_*(k,t)\delta_*^*(k',t)\rangle$ to obtain $P_*(k,t)$

Small-scale suppression of stream power

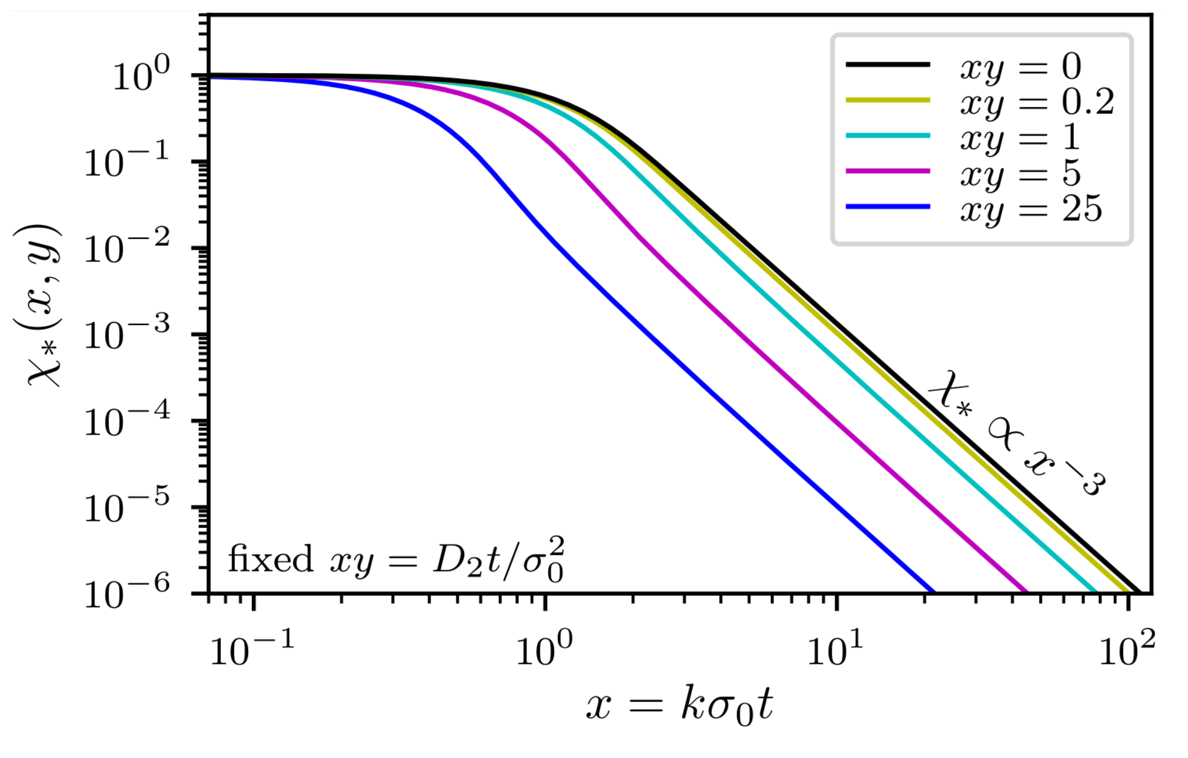
…after much work we obtain:

$$P_*(k,t) = \chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)\frac{k^2t^2}{3}P_{\Delta v}(k,t)$$

Stream density power spectrum

$= P_*(k,t)$ if stream's velocity dispersion is neglected

"Transfer function" χ_* describes how the stream's velocity dispersion suppresses small-scale power.
[χ_* depends on velocity dispersion's initial value σ_0 and growth rate D.]



Toward a realistic stellar stream

We have the connection between stream power spectrum and perturber power spectrum!

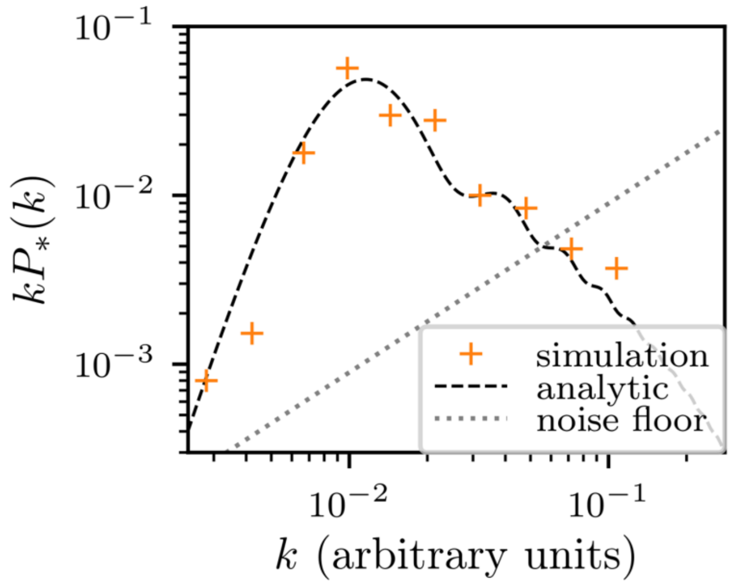
(e.g., dark matter substructure)

$$\boxed{P_*(k,t)} = \chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)\frac{k^2t^2}{3}P_{\Delta v}(k,t)$$

$$P_{\Delta v}(k,t) = 16\pi^4 G^2\bar{\rho}^2 k^2 t\int_k^\infty \frac{\mathrm{d}q}{q}\boxed{\frac{\mathcal{P}(q)}{q^6}}\int \mathrm{d}^3\boldsymbol{u}\frac{f(\boldsymbol{u})}{u}\theta_H(qu-kv)$$

Analytic expressions were validated against idealized particle simulations:

- Start with a "stream" consisting of a periodic line of stars with some velocity dispersion
- Subject stars to random impulses due to passing Plummer spheres [which have an associated power spectrum]
- Compare stream power spectrum to analytic prediction



Toward a realistic stellar stream

We have the connection between stream power spectrum and perturber power spectrum!

(e.g., dark matter substructure)

$$P_*(k,t) = \chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)\frac{k^2t^2}{3}P_{\Delta v}(k,t)$$

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…at least for our simplified picture.

Practical complications

- Orbital dynamics: the connection between Δv and Δx (in the stream frame)
- Stream grows outward from, and is continuously sourced by, progenitor

Can address at an approximate level

Substructure power spectrum from subhalos

We now have stream power $P_*(k)$ as a function of substructure power $\mathcal{P}(q)$.

Other stellar stream treatments consider subhalo populations instead.
These are related:

Substructure power spectrum (dimensionless form)

Subhalo mass function

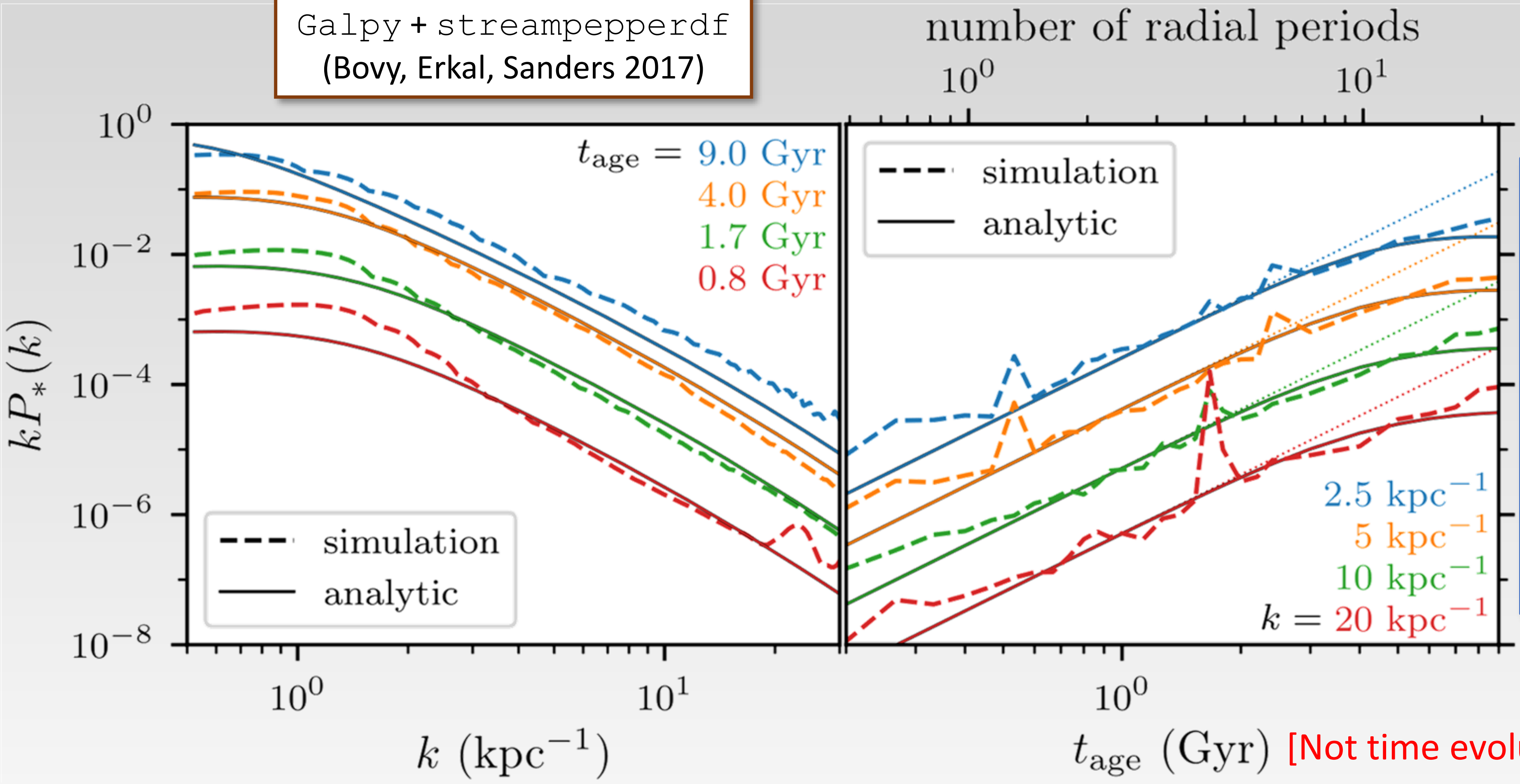
$$\mathcal{P}(q) = \frac{q^3}{2\pi^2}\frac{1}{\bar{\rho}^2}\int_0^\infty \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M}\tilde{\rho}_M(q)$$

Fourier-transformed density profile for a subhalo of mass M

(we assume subhalo positions are uncorrelated)

Analytic prediction vs simulation

Dashed curves use
`Galpy` + `streampepperdf`
(Bovy, Erkal, Sanders 2017)



[Not time evolution!]

Matches numerical results except at:

- Small k (outside diffusion regime)
- Low t_{age} (complex orbital dynamics)
- High t_{age} ($Dt > \sigma_0^2$ here) [dotted: $D = 0$]

(We consider a model of the GD-1 stream perturbed by a simplified model of CDM substructure)

Implications

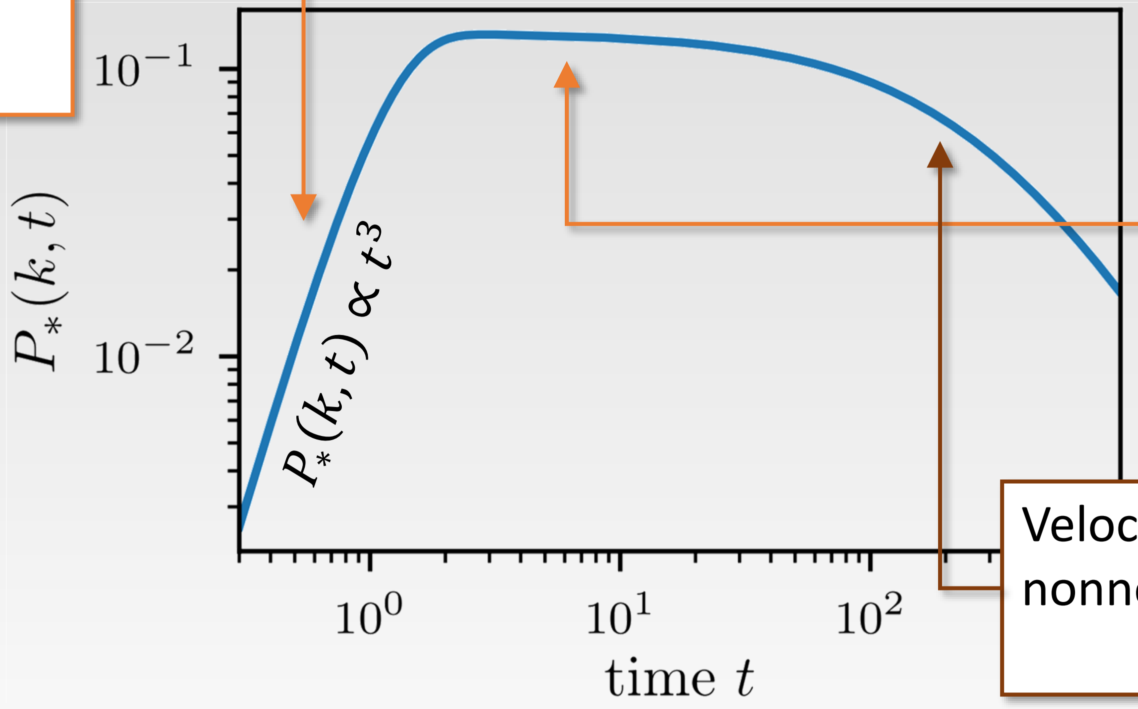
General time evolution of stream perturbations

$$P_*(k,t) = \underbrace{\chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)}_{\propto\, t^0,\ t^{-3},\ \text{or}\ t^{-4.5}\ \text{in different regimes}} \frac{k^2t^2}{3} \underbrace{P_{\Delta v}(k,t)}_{\propto\, t}$$

Velocity dispersion negligible
$\sigma < \frac{1}{kt}$

Velocity dispersion nonnegligible but constant
$\sigma \simeq \sigma_0 \gg \frac{1}{kt}$
(perturbations in steady state between injection and suppression)

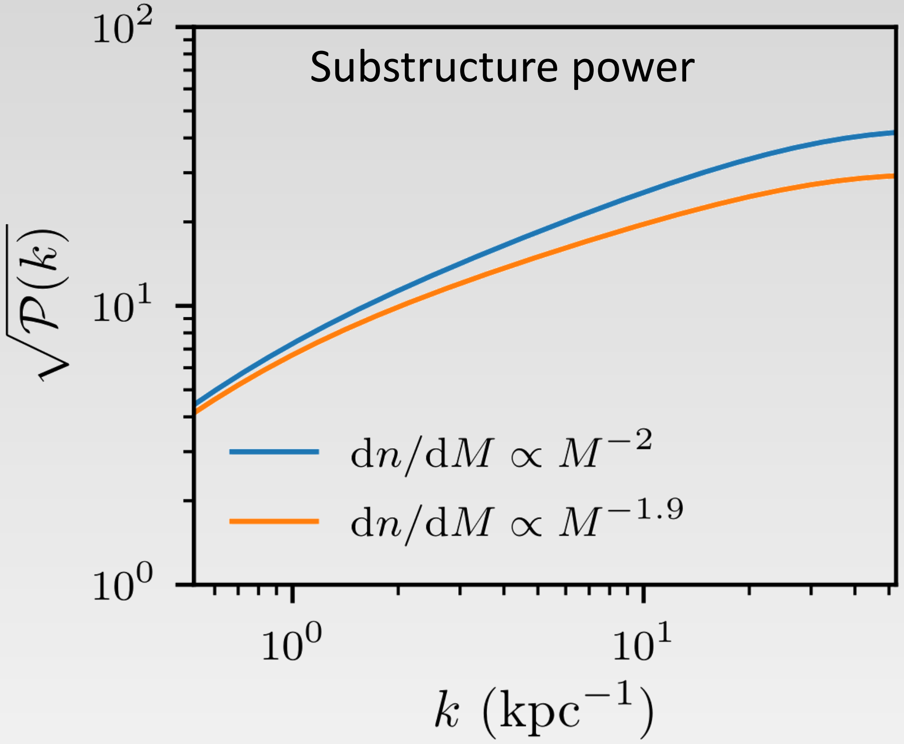
Velocity dispersion nonnegligible and growing
$Dt > \sigma_0^2$



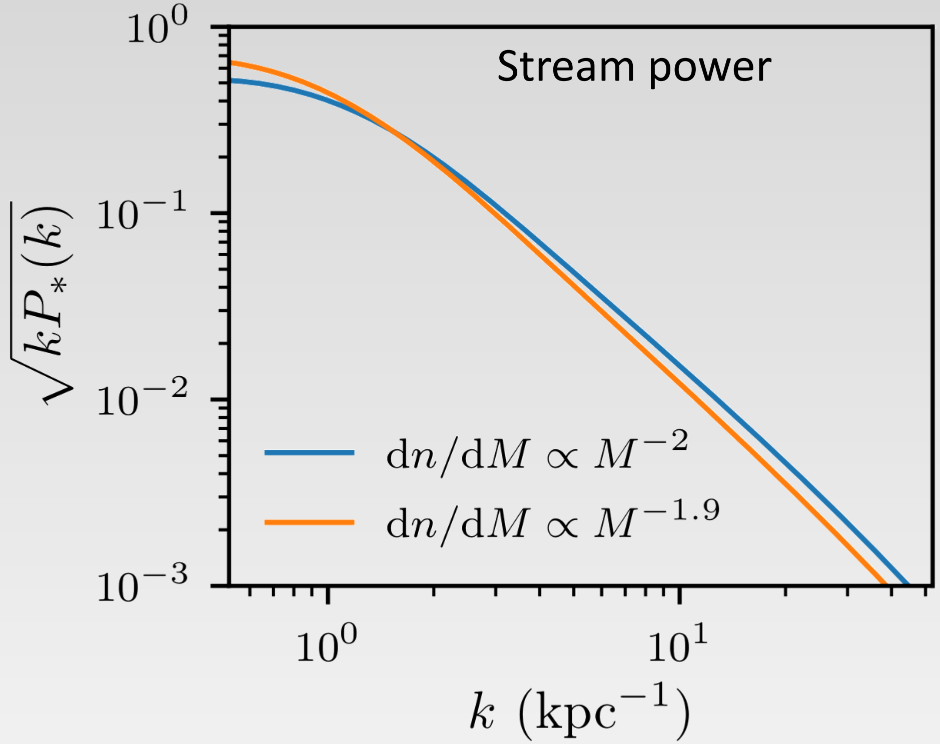
position along stream

Implications

Impact of subhalo mass function



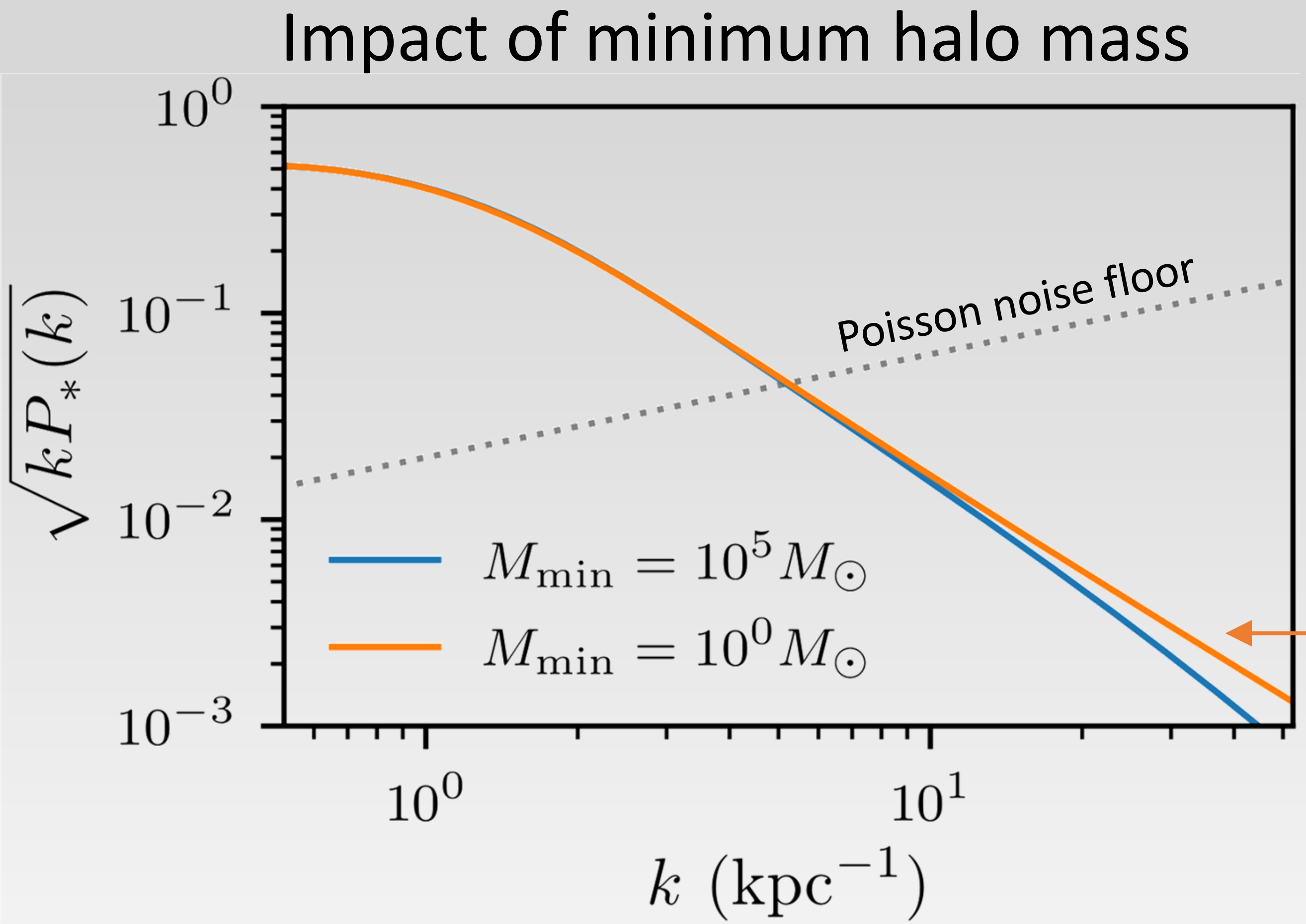
$\Longrightarrow$



More small-scale substructure power → more small-scale stream power (unsurprisingly)

[Although greater induced velocity dispersion partially compensates]

Implications



Increased stream power due to subhalos with $M < 10^5 M_\odot$ lies below noise floor (for GD-1)

Prospects are poor for probing halos below $10^5 M_\odot$ (at least with GD-1 density). But $10^5 M_\odot$ is still valuable!

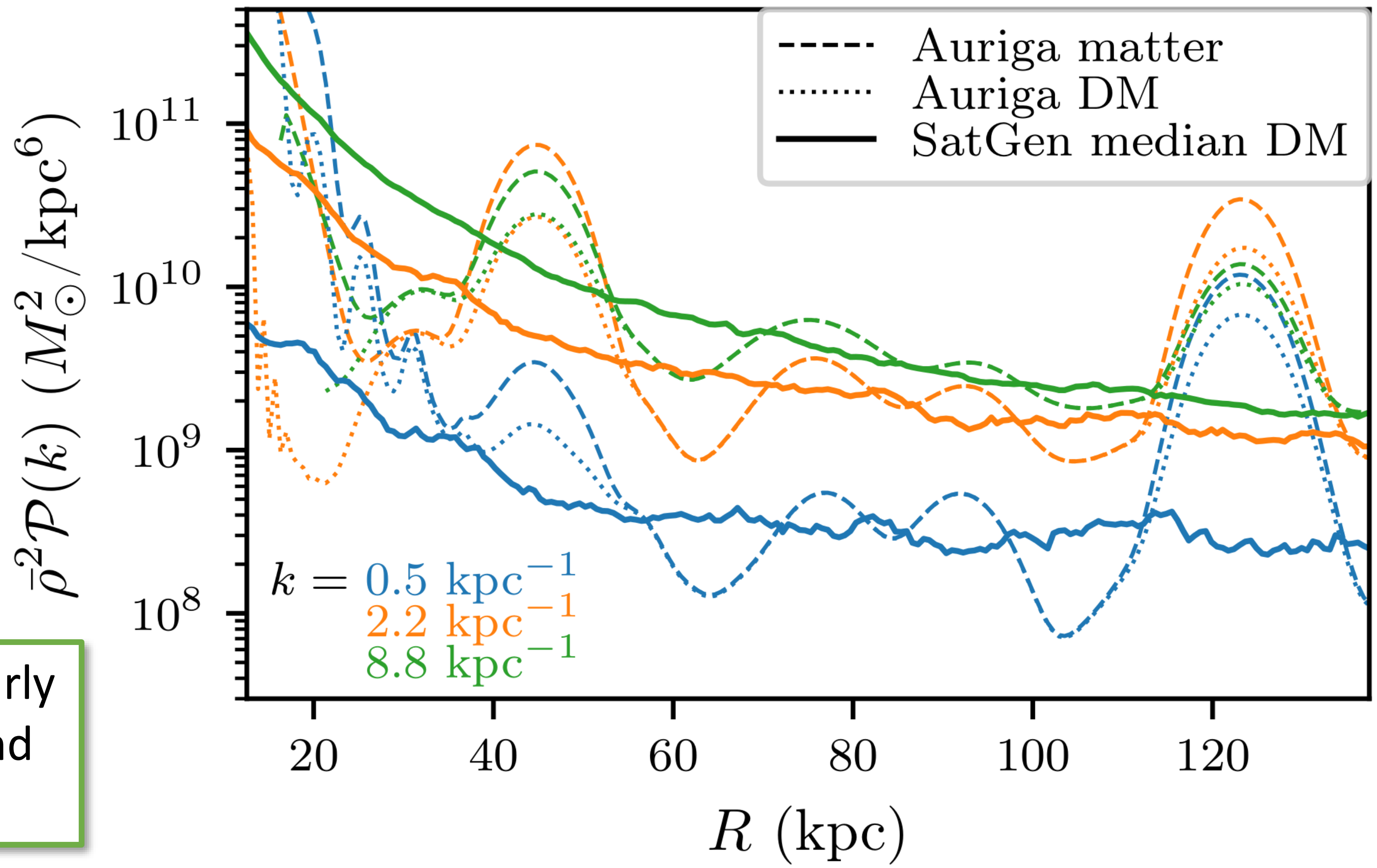
Implications

Where to look for perturbed streams?

Simply look at the density power spectrum $\bar{\rho}^2\mathcal{P}(k)$!

Plot: $\bar{\rho}^2\mathcal{P}(k)$ about a 10^{12} $\mathrm{M}_\odot$ halo in:

- **Auriga** hydrodynamic simulation [1 halo] (Grand et al 2021)
- **SatGen** semianalytic subhalo model [120 halos] (Green et al 2021, 2022)



DM substructure content nearly uniform past $R \sim 40$ kpc, and no baryonic structure!

Summary

Stellar streams retain a sharp memory of past gravitational perturbations.
Thus, they serve as a purely gravitational probe of dark matter.

The power spectrum of a stellar stream's density can be analytically related to that of the substructure background:

$$P_*(k,t) = \chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)\frac{k^2t^2}{3}P_{\Delta v}(k,t)$$

Stream power

Substructure power

$$P_{\Delta v}(k,t) = 16\pi^4 G^2\bar{\rho}^2k^2t\int_k^\infty \frac{\mathrm{d}q}{q}\frac{\mathcal{P}(q)}{q^6}\int \mathrm{d}^3\boldsymbol{u}\frac{f(\boldsymbol{u})}{u}\theta_H(qu-kv)$$

Our premise was to write $\Delta\boldsymbol{v}$ as a k-space integral over the perturber environment…

$$\Delta\boldsymbol{v}(\boldsymbol{r}) = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\delta(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\boldsymbol{V}^*(\boldsymbol{k}|\boldsymbol{u},t), \quad \boldsymbol{V}(\boldsymbol{k}|\boldsymbol{u},t) \equiv 8\pi\mathrm{i}G\bar{\rho}\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{u}t/2}\frac{\sin(\boldsymbol{k}\cdot\boldsymbol{u}\,t/2)}{\boldsymbol{k}\cdot\boldsymbol{u}}\frac{\boldsymbol{k}}{k^2}$$

…and thereby relate the statistics of $\Delta\boldsymbol{v}$ to the environment's power spectrum.

This method may be applied to other dynamical probes of dark matter as well!